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Five Hundred Years of Bookkeeping
A Portrait of Luca Pacioli

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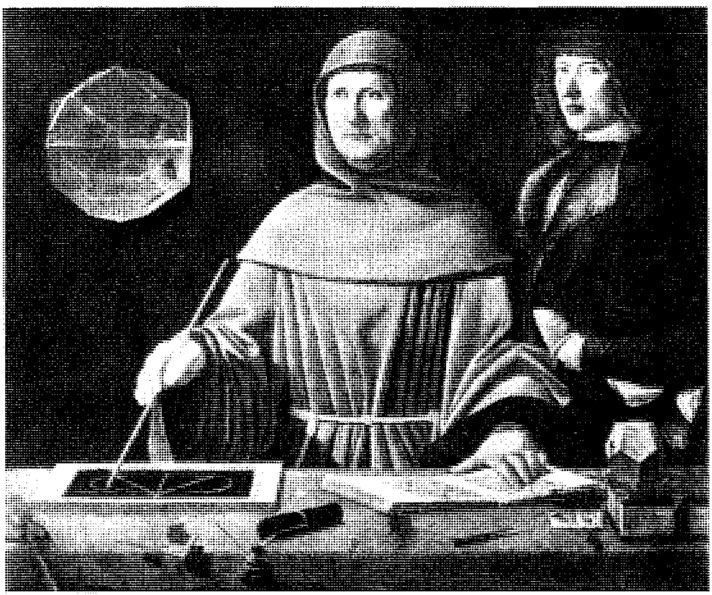

Pacioli's portrait,
Painting by Jacobo de Barbari (1495)
(Museo Nazionale di Capodimonte te Napels).

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I. INTRODUCTION

In 1494 the *"Summa de arithmetica, geometria, proportioni et proportionalita"* (Review of arithmetic, geometry, and proportions) was published in Venice. This book is the first[1] printed important book on algebra and was the best-known book on algebra in the early Renaissance. As a Summa, it aimed to summarize the mathematical knowledge of those days. Its author, Luca Pacioli, was a Franciscan monk who during his life enjoyed great fame as a professor. He taught at the best Italian universities and was very much in demand in the most distinguished intellectual, artistic, religious, and court circles of the time.

The eleventh *"tractatus"* of the nineth *"distinctio"* of the Summa, entitled *"Particularis de computis et scripturis"* (About accounts and other writings), provides a detailed discription of Venetian bookkeeping. This tractatus is known as the first printed essay on double-entry bookkeeping and was a direct base of some widespread works on mercantile accounting. As the tractatus deeply influenced the development of accounting, Luca Pacioli is rightly called the founding father of this discipline.

II. PACIOLI'S LIFE (1445-1517)[2]

Luca Pacioli[3] was born in 1445 in Borgo San Sepulcro, a small Tuscan town and belonged, being the son of Bartholomeus Pacioli, to a middle class family. His first teacher was no less a person than the painter Piero della Francesca (1415/1420-1492), who, typically for Italian Humanism, masterly connected mathematics, science and art. In 1464 Luca Pacioli became employed as a private teacher by a rich Venetian merchant by the name of Antonio de Rompiasi. Together with Rompiasi's sons he attended the lectures of the mathematician Domenico Bragadino in the Scuolo di Rialto, a school of great importance for the history of Aristotelianism. Most probably he also worked as Rompiasi's bookkeeper. In 1470 Pacioli stayed in Rome at the house of the famous architect, philosopher and mathematician Leon Battista Alberti (1404-1472). This move to Rome was advised by his teacher Piero, who had worked together with Alberti in the church of San Francesco in Rimini during the fifties. In 1473 Pacioli became a franciscan Minor. As Frater Lucas de Borgo San Sepulcro (and from 1486 on, after some theological and philosophical studies, as Magister), he visited the main courts and lectured mathematics at various Italian universities such as Perugia, Florence, Rome, and Napels, where he also taught military science. In 1494 he went to Venice to prepare the publication of the Summa. Pacioli's opus magnum, to be seen as the result of his lectures, was edited by Paganino de Paganini (November 10, 1494). In 1496 he was the first occupant of a chair of mathematics founded by duke Ludovico Sforza at Milan. Like the Malatesta and the Montefeltro dynasties in Rimini and Urbino, the Sforza family fits in the tradition of Italian maecenatism. In Milan, he became a friend of Leonardo da Vinci (1452-1519). After the fall of Ludovico (1499), Pacioli and da Vinci moved together to Florence. In 1509 Pacioli returned to Venice to publish the *"De divina proportione"*, his second major book (with illustrations of da Vinci), and the *"Euclidis opera"*, a revision of Campano's translation (13th century) from Arabic into Latin. His career ended with the highest honor, namely the appointment by Pope Leo X in 1514 as professor of mathematics in the Sapienza at Rome, a position of the highest ranking. After a year Pacioli returned to Borgo San Sepulcro, and died on June 18, 1517.

We close this section with a description of Pacioli's portrait in the Museo Nazionale di Capodimonte at Napels (see title page)[4]. This portrait reflects Pacioli's mathematical curriculum. It shows Pacioli standing behind a table and wearing the habit of a Franciscan. He draws a construction on a board, the edge of which bears the name "Euclides". His left hand rests upon a page of an open book. This book may be his Summa (Cantor (1900), p336, de Waal (1927), p53)) or a copy of Euclid (Littleton and Yamey (1956), p181). Upon the table rest the instruments of a mathematician: a sponge, a protractor, a pen, a case, a piece of chalk, and compasses. In the right corner of the table there is a dodecahedron resting upon a book bearing Pacioli's initials. An icosahexahedron (a convex solid consisting of 18 squares and 8 triangles) suspends at the left of the painting[5]. The identity of the young man at the right is uncertain. Volmer (1993) recognizes the eternal student instructed by Pacioli.

III. LUCA PACIOLI AS A MATHEMATICIAN

Pacioli's main mathematical work[6], to wit, his Summa and his Divina proportione, concentrates on arithmetic, algebra, and geometry.

The Summa, as already indicated, is a compilation of the works of previous authors, such as Leonardo de Pisa (known as Fibonacci), Boethius, Sacrobosco, Ptolemy, Euclid and Archimedes. According to Volmer (1993) as much as three quarters of the Summa was taken from Fibonacci[7]. The part of the Summa dealing with arithmetic and algebra is strongly based upon Fibonacci's "Liber abaci". This Liber abaci (1202), influenced by the algebras[8] of Arab mathematicians such as al-ḵuwārizmī (ca.780 - 850), very much enhanced the introduction of the Arabic numerals into Europe. The part of the Summa dealing with geometry is considered a poor summary of the "Elements" of Euclid (ca.300 A.C.) and the "Practical geometriae" of Fibonacci (1220) (e.g. Smith (1951), p253). Euclid, a monument in the history of mathematics, is one of the first writers who gave a systematic exposition of the leading propositions of elementary metrical geometry.

Although the Summa exhibits almost no originality, it is generally considered as an important work. This, mainly because of its wide circulation (it was written in the vernacular, and it was a printed book). Pacioli's second major book, the Divina proportione, is entirely devoted to geometry. This work of Pacioli is of more merit from the standpoint of geometry than the Summa, but was far less popular. In the remainder of this section both works of Pacioli are discussed. In addition, to give a flavour of mathematics in the 15th century, some topics Pacioli dealt with are presented.

We start with Pacioli's arithmetic and algebra. The first nine distinctions of the Summa study this area of mathematics and cover the following subjects. The section on arithmetic provides rules for the processes of addition, subtraction, multiplication, division, and a method for extracting square roots[9]. The section on algebra includes the standard solution of linear and quadratic equations. Pacioli provides a classification of cubic equations, but adds that their solution appears to be impossible just like the quadrature of the circle[10].

In fact, Pacioli presents the arithmetic and algebra as developed through contributions of Moorish scholars in Spain during the Arab domination, and later on through European commercial relations with the Arab world. Nevertheless, the Summa contributed to the evolution from rhetorical to syncopated algebra. As such, Pacioli denotes addition by p or $\bar{p}$ (piu), equality by *ae* (aequalis), and sometimes subtraction by $\bar{m}$ (meno) or by *de* (demptus). Following Fibonacci and the Arabs, he calls the unknown quantity the "thing" and denotes it by *co* (cosa) or by *R* (res). Therefore, Howard Eves ((1969), p213) captures the significance of the Summa as follows: "The Summa contains little of importance not found in Fibonacci's Liber abaci but does employ a superior notation."

Now, let us single out a few algebraic problems that occur in this first part of the Summa. A first problem is a very classical one and illustrates the so called "method of false position" to solve a linear equation[11]. Pacioli formulates the question referring to a merchant. Suppose that a merchant spends a quarter of his capital in Pisa and a fifth in Venice, that he receives on these transactions 180 ducats, and that he holds 224 ducats in the hand (hence, 44 belong to the original capital). What was his original capital? To solve this problem by way of "false position", one tries any number, e.g. an original amount equal to 100 ducats. Then, if he spent 25 and 20 ducats at Pisa and Venice, he would have 55 ducats left (instead of 44). Since the ratio of the right amount to 100 is as the ratio 44 to 55, the original capital was 80 ducats.

A second problem, nowadays known as the "problème des partis" of Chevalier de Méré, became central in the development of probability theory. Pacioli is one of the first writers who introduced it into a work on mathematics. The problem is as follows[12]. "Two persons, *a* and *b* are playing a game of chance (both players have an equal probability to win). The game is repeated, and a score reflects the number of victories. Let 6 points be sufficient to win the stakes (to be placed in advance). How to divide the stakes if the game is interrupted after a score of 5 points for *a* against 2 for *b*?

The right answer to this question was given by Blaise Pascal (1623-1662) in his 1654-correspondence with Pierre de Fermat. Pascal's reasoning is as follows. If both players have 5 points, the stakes are equally divided. Next, consider a score of 5-4. After playing the 10th repetition, the score becomes 6-4 and *a* gains all, or the score becomes 5-5 and the stakes are divided equally. Hence, player *a* is certain of 1/2, and may gain an extra 1/2. The share for player *a* becomes $1/2 + 1/2 \bullet 1/2 = 3/4$. Analogously, a score of 5-3 (resp., 5-2) results in a share of $3/4 + 1/2 \bullet 1/4 = 7/8$ (resp., $7/8 + 1/2 \bullet 1/8 = 15/16$) for player *a*.

Pacioli, totally unaware of the difficulties related to the notions of "chance" and "probability", solves this exercise incorrectly and suggests a division of 5/7 against 2/7 (Cantor (1900), p327).

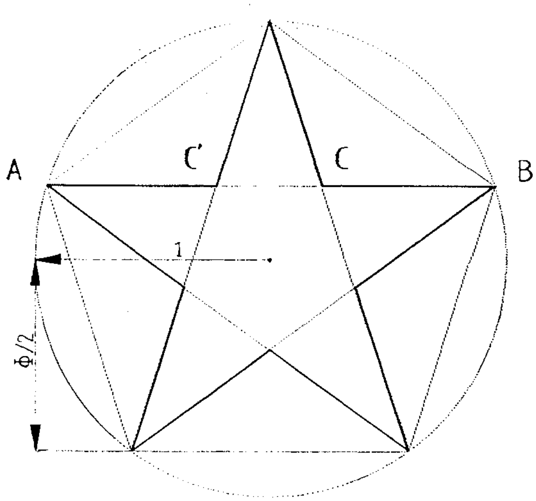
Next, we discuss some of Pacioli's geometrical works, to wit, the second part of the Summa and the Divina proportione. In his Summa, Pacioli exploits the possibility to employ algebra to solve geometrical problems. For example, in order to solve a triangle when the inscribed circle together with the segments into which one side is divided by the point of contact are known, Pacioli elaborates a geometrical construction and converts the problem to finding the roots of a quadratic equation. This approach, quoted by Geronimo Cardano (1501 - 1576) as incomparably simple and excellent to solve the problem at hand, is now experienced as involved and inelegant (Ball (1908), p212)[13].

The Divina proportione deals with regular polygons, the five Platonic solids, and with the ratio later known as "the golden section".

The golden section, already studied by the Greek mathematician Eudoxos (408-355 A.C.), is experienced as a fundamental ratio in the art of painting, architecture, music, etc.. The golden section cuts a line in extreme and mean ratio, i.e. a line AB is split up in AC and CB such that $AC:AB = CB:AC$. Set the length AC equal to 1 and denote Φ the length AB, then $1/\Phi = \Phi - 1$. Hence, $\Phi = (\sqrt{5} + 1)/2$. Note that Φ can be constructed with the Euclidean tools, i.e. the straightedge and compasses[14]: $\sqrt{5}/2$ is the hypotenuse of a right triangle with legs 1 and 1/2. Figure 1 shows the golden section in the construction of a star pentagon.

FIGURE 1

The star pentagon
C (resp. C') is the golden section of AB (resp. AC).



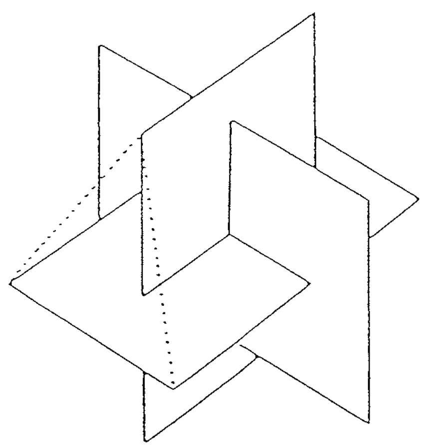
The regular solids, i.e. convex polyhedra consisting of congruent regular polygons and having congruent polyhedral angles, are extensively dealt with. The cube, made up of six equally large squares, is an example. Plato (428/427-347 A.C.) associated these solids with the four primal elements: the cube with earth, the tetrahedron (4 triangles) with fire, the octahedron (8 triangles) with air, and the icosahedron (20 triangles) with water. The fifth regular solid, i.e. the dodecahedron (12 pentagons), symbolizes the enveloping universe:

"There still remained one construction, the fifth; and the god used it for the whole," (Plato's Timaeus, 55c; see Cornford (1956)).

The study of these polyhedra has led to some important developments in the theory of groups, discrete mathematics, and in topology[15].

FIGURE 2

The skeleton of the icosahedron and the dodecahedron (each rectangle has size 1 x Φ).



Pacioli offers the following construction. Put three golden rectangles as in Figure 2 and focus on the 12 vertices. The distance between any pair of neighbouring points is equal to 1. Hence, these 12 points coincide with the 12 vertices of the icosahedron. Furthermore, one can construct the dodecahedron by placing 12 pentagons the centers of which coincide with the 12 points. According to this construction, the icosahedron is placed inside the dodecahedron: water as a part of the universe.

IV. TRACTATUS XI

The *"Summa de arithematica, geometria, proportioni et proportionalita"* counts more than three hundred folios. The title does not reflect the inclusion of a chapter on accounting. However, thirteen folios (numbers 198-210) are dedicated to Venetian bookkeeping. It is not at all surprising to find a treatise of mercantile procedures in a book on mathematics. At the time, mercantile arithmetic was an established part of mathematics, and its teachers were mathematicians. The great intellectual Piero della Francesca, for example, did not consider it unworthy of his talents to deal with merchants' questions of abacus, and wrote the "Trattato d'abaco". Pacioli also admired the extensive knowledge and high qualities required by whoever engages in trade: "You need to know more to be a good merchant than to be a doctor of law" (Summa, folio 199, verso). The Flemish mathematician Simon Stevin (1548-1620), originally a merchant's clerk in Antwerp, places bookkeeping at the same level as the Artes liberales (Vrije consten): "Coopmans bouckhouding is een const, welcke ick, hoewelse in Barbaro soeculo ghevonden schijnt, weerdich acht onder de Vrije gerekent te worden" (de Waal (1927), p279, note 1).

In the remainder of this section, Venetian bookkeeping as it appears in Pacioli's eleventh tractatus "Particularis de computis et scripturis", is briefly explained[16].

According to Pacioli, accounting is an ad hoc ordering system devised by the merchant. Its regular use provides the merchant with continued information about his business, and allows him to evaluate how things are going and to act accordingly. Pacioli recommends the Venetian method of double-entry bookkeeping above all others. Three major books are at the direct basis of this system: the memorandum (memoriale), the journal (giornale), and the ledger (quaderno). The latter book is considered as the central one and is accompanied by an alphabetical index. The use of several other books is also suggested in Pacioli's treatise, such as: an inventory book and a receipt and/or payment book.

Pacioli describes the accounting procedures to be adopted right from the start up of a business. Before starting a business, it is considered as necessary to make an *inventory* (inventario) which comprises all the assets and debts. The inventory always relates to one single day and is to be written on a separate folio or in a specific book. In terms of today's jargon, this beginning inventory corresponds with an opening statement of financial position, or simply an opening balance sheet. Apart from furniture, all the main categories of goods are listed in the inventory: cash money, jewels, valuables, clothes, furnishings, merchandises, buildings, estates, deposits, borrowings, debtors and creditors. By posting the inventory entries to the journal and then to the ledger, Pacioli initiates the reader into the accounting mechanics of double-entry: each journal entry gives rise to two entries in the ledger, once as a debit and once as a credit[17]. Note, however, that Pacioli only refers to the inventory (which can be seen as a kind of balance sheet) at the beginning of the merchant's operations. At the time, it was not common to make an inventory, i.e. to draw a balance sheet, on a regular basis.

The subsequent recording of transactions is described as follows. First, there is the *memorandum,* which is a kind of diary and which only serves as a first rough draft. The events are recorded in the memorandum by anyone in the business who executes a transaction; this is done in chronological order and is accompanied by some narrative details. From the memorandum the bookkeeper is to record the transactions into the *journal.* In the journal the events are converted in debit and credit amounts, and all amounts are to be expressed in the same type of money[18]. In a journal entry, "Per" (from) is used to indicate the account that is affected at the debit side, and "A" (to) is used to indicate the account that is affected at the credit side. In summary, in the journal the opening inventory and all subsequent changes therein are recorded. Since no closing (i.e. end-of-period) entries are included in the journal, this book is to be considered as a technical aid in preparing the accounts in the ledger.

All journal entries are posted to the corresponding accounts in the ledger. Each account in the ledger took both sides of a double page. Debits were posted on the left hand side, and credits on the right hand side. Note also that Pacioli uses the terms "deve avere" (has to have) when referring to assets or asset accounts, and "deve dare" (has to give) when referring to liabilities or liability accounts. Pacioli explicitly states that the ledger should at all times be in equilibrium, and should this not be the case, the accounting system is in error. He further recommends to check the equilibrium on a regular basis. For this purpose, he suggests a procedure *outside* the accounting system, that is to list the totals of debits and credits per account and to make the sum of all debits and credits across accounts, which he calls the *Summa Summarum.* What Pacioli actually suggests is to make a (interim) trial balance. Recall however that this exercise is not necessarily related to the process of closing the books (although it is adopted then as well), as it merely serves as a check.

As to the question when and how the books should be closed, Pacioli's approach is incomplete in the sense that the procedure is entirely done outside the accounting system. Pacioli makes two suggestions as to when the books should be closed: either when the books are full, or at the end of the year. The procedure of closing the books consists of four steps. The procedure is initiated by ticking and checking the entries. Second, the asset and liability accounts (that is the inventory accounts) in the ledger are closed *directly.* This implies that per account the balance between debit and credit amounts is computed and recorded at the side with the smaller total. This balance then serves as the opening balance in the new ledger. Next, the nominal and expense accounts are closed in the same (direct) way, but here the balances are transferred to the profit and loss account, and subsequently profit or loss is transferred to the capital account. Finally, the capital account is closed and its balance transferred to the new ledger. Before posting the closing balances to the new ledger, Pacioli recommends to compute the Summa Summarum (see before) as a check on the equilibrium in the accounts.

An important observation is that Pacioli does not make a direct link between closure of the accounts to open a new book, and drawing a balance sheet. Furthermore, the computation of profit or loss is not only done when the books are closed, but much more frequently. Again, this is done outside the accounting system on a separate sheet by listing all the debit and credit amounts for the nominal and expense accounts and herefrom computing the balance. In the entire closing procedure, it is striking that no balancing accounts are used, since the accounts are closed directly. As a result the journal cannot serve to check the correctness (i.e. the equilibrium) of the accounting system. A similar "direct" approach is promoted by Stevin. Ympijn is the first author to introduce the use of balancing accounts in the closing procedure. Both authors published on double-entry bookkeeping after Pacioli (see Section V).

Some other - less technical - features in Pacioli's treatise are worth mentioning. It is, for example, recommended to have books authenticated by the official merchants' organisation in order to avoid fraudulent practises. Also, Pacioli almost always uses Arabic numerals to write amounts in the money column. Roman numerals are used to write the year at the top of each folio of the ledger. Finally, to distinguish successive sets of books, Pacioli recommends to mark the books of the first set with the sign of a cross, those of the second set with an A, of the third with a B, and so on.

V. ACCOUNTING BEFORE AND AFTER PACIOLI

Pacioli does not refer to the origins of double-entry bookkeeping. He does, however, by no means claim to have invented the system, explicitly stating that his work is a reflection of Venetian accounting rules as developed through international trade. During the eleventh and twelfth century the Italian city-states such as Genoa, Florence, and Venice were the leading trade centers. The oriental trade routes, extended during the crusades, turned Venice into a world market for oriental products (spices, sugar, different kinds of wood, cotton, silk, perfumes, ivory, glasswork, etc.). Most likely, the Italians picked up their knowledge of double-entry bookkeeping at Alexandria, Constantinople, or some other eastern city (Colt (1844),p230). Recent investigations are supportive of this thesis. Albraiki (1994), for example, proves that during the early years of the Mamluk period (1250-1517) double-entry bookkeeping was already in use in Egypt and Syria. It further appears that the Old Cairo geniza collection!)[19] contains a fragment (dated ca. 1080) in the form of a journal and a four page account (dated 1134) listing both credits and debits (Scorgie (1994)).

Most accounting historians, however, do not accept double registration of a transaction, once as a debit and once as a credit, as a sufficient (albeit necessary) condition to meet the requirements of a double-entry accounting system. de Roover ((1952), p114), for example, states: "A necessary *prerequisite* is that all transactions be recorded twice, once on the debit and once on the credit side. If this requirement is not fulfilled there is, by definition, no double entry. This principle involves the existence of an integrated system of accounts, both real and nominal, so that the books will balance in the end, record changes in the owner's equity and permit the determination of profit or loss" (italics added). Kam ((1986), p29) defends the opinion that the key aspects of double-entry relate to the duality of accounts and the duality of classifications (i.e. assets versus equities) and to their equilibrium.

From a survey of accounting development in the Italian city states in the pre-Pacioli era by de Roover (1952), it can be concluded that at least "some" principles of double-entry bookkeeping were practised in Florence in the late 13th century. Surviving fragments of account books of Rinieri Fini (dated 1296-1305) and Forolfi (dated 1299-1300), both merchant-bankers, indicate that each entry, albeit still in paragraph form, gives a cross reference to a corresponding (earlier or later) debit or credit. de Roover strongly rejects that this distinctive feature (as compared to other systems at the time) meets the requirements of double-entry. The rationale is that there is no indication of a procedure to close the books, and - in our opinion more importantly - there is no evidence that at the end a balance *can* be drawn showing the assets, liabilities and the owner's equity.

The oldest discovered record of a complete double-entry system is the Messari (treasurers) accounts of the city of Genoa in 1340. These books do not only contain debits and credits journalised in a bilateral form, but also each transaction is recorded twice in the ledger. The ledger of 1340 further contains balances carried forward from a preceding one of the year 1339. Because of this, the Messari accounts enjoy general recognition as a double-entry system. Unfortunately, no records prior to 1340 were found in the Genoase archives. Apart from the development of the principles of double-entry bookkeeping through Italian accounting practice, one other event in the pre-Pacioli era is worth mentioning. In 1458, and hence prior to the publication of the Summa, Benedetto Cotrugli wrote a chapter, entitled "Dell'ordine di tenere le scritture mercantilmente" in a general book on merchant practices (entitled "Della mercatura e del mercante perfetto"). This book remained, however, unpublished until 1573. Although various accounting historians categorise Cotrugli's chapter as containing elements of double-entry bookkeeping, there are major quality differences with Pacioli's work. First, only a few folio's (one chapter out of fifty in a book containing 212 folio's on trade in general) are dedicated to bookkeeping. Second, the influence of Pacioli's work over the years has been far more reaching than Cotrugli's (even after 1573). de Waal (1927) considers Cotrugli's work as a general description of trade practice at the time, and concludes that most likely by the second half of the 15th century double-entry principles were extant in Italian city states.

The spread of the Italian accounting rules over the rest of Europe and thence further afield, was the result of treatises, some of them strongly based on Pacioli's work, describing and explaining the system and its practice. The "Quaderno doppio" (Venice (1534)) of Domenico Manzoni da Oderzo was one of the first reproductions of Pacioli's "De computis". This work, important because of elaborate examples, was very popular and widespread among merchants: it enjoyed no less than seven editions between 1534 and 1574. Other books that are directly or indirectly based on Pacioli's work are Hugh Oldcastle's "Profitable treatyce" (London (1543)), a translation of Pacioli's tractatus (Lanero (1994)), and Wolfgang Schweicker's "Zwifach Buchhalten" (Double-entry bookkeeping, Neurenberg (1549)), a translation of the Quaderno doppio.

The first printed treatise on Italian bookkeeping in the Dutch language was provided by Jan Ympijn Christoffels, the founder of the socalled Flemish school (ten Have (1973), p69). His "Nieuwe instructie" (New instructions, Antwerp 1543), a rewrite of Pacioli's tractatus, was also translated into French (1543) and English (1547). Simon Stevin was the last author of this Flemish school. His "Wiskonstige ghedachtenissen" (Mathematical memoirs, Leiden (1608)) contains about 100 pages on bookkeeping. In addition, Stevin gave new insights in the general principles behind accounting. de Waal ((1927), p289) states that the development of the theoretical side of accounting until as late as the 19th century was based on the writings of Luca Pacioli and Simon Stevin.

In the post-Pacioli era, thus, relatively few changes to the double-entry accounting system emerged until the 19th century. The industrial revolution and the rise of capitalism propelled the further development of accounting systems and accounting theory in the 19th and 20th century. Aggregation of capital and labour in factories where steam and other power sources were applied to produce standardised goods on a large scale led to dissatisfaction with some aspects of merchant double-entry bookkeeping (Mathews and Perera (1991), p19). Problems associated with depreciation of machinery, allocation of fixed costs and overheads and some inventory costing issues were new at the time, and had not existed in the days of craft industries and owner managers. As a result, "cost accounting" issues received considerable attention.

The need for large amounts of capital to finance industrialisation resulted in pressures to change the legal framework in which companies operated (Goldberg (1949), p22). Separation of ownership and control was first established in the U.K. in the middle of the 19th century by way of Joint Stock Companies and Limited Liability Companies. This resulted in a split of accounting systems for internal (i.e. *managerial accounting*) and external (i.e. *financial accounting*) purposes, and subsequently also in accounting and disclosure regulations and a growing need for independent attestation of external accounts by *auditors.*

Nowadays accounting research and practice continues to develop in various directions and in correspondence to economical, regulatory, and social conditions. Nevertheless, Pacioli is not forgotten, which may be concluded from the foreword of the Programme book of the 17th annual congress of the European Accounting Association (Venice, April 6-8, 1994): "The choice of Venice also has a strong symbolic meaning for all accountants. As is well known, it was in Venice that in 1494 Fra' Luca Paciolo published his "Summa", the first printed book on double-entry bookkeeping. The Venice Congress, therefore, coincides with the five-hundredth anniversary of the emergence of our field."

VI. CONCLUSION

Pacioli wrote the first printed mathematical encyclopedia, known as the Summa. This work set the boundaries of contemporary knowledge, and drew a framework for major advances in algebra which would take place during the next century. Thanks to the Summa, Pacioli has gained a place in the history of mathematics. By including a treatise on bookkeeping, Pacioli is considered the father of accounting. As he lived and taught in the Italian city-states during the early Renaissance, Luca Pacioli was in a fortunate position. Libraries had been enriched with manuscripts of ancient Greek writers as a result of the fall of Constantinople to the Turks in 1453, and the related flow of refugees into Italy. Further, thanks to the invention of printing knowledge disseminated at an unprecedented rate. And finally, Pacioli's experience in bookkeeping turned him into an eligible candidate to organize and to structure a large part of mercantile arithmetic which had been developed through the intense commercial activity in Italy.

We end this paper by quoting the inscription on the memorial stone placed in Pacioli's house of birth in Borgo San Sepolcro in 1878. The text reflects the essential significance of Pacioli[20]:"For Luca Pacioli, who had da Vinci and Alberti as friends and advisers, who turned algebra into a science, and applied it to geometry, who lectured in double-entry bookkeeping, whose work was the base and the norm for later mathematical research; for this great fellow citizen, the people of San Sepulcro, ashamed of 370 years of silence, have placed this stone, 1878."

NOTES

1. The art of printing dates from the beginning of the 15th century (Gutenberg's bible dates from 1456). Earlier books on mathematics in printing form, such as the "Sphaera" by Sacrobosco (Ferrara, 1478), were more elementary.
2. This section follows de Waal (1927), Littleton and Yamey (1956), and Volmer (1993).
3. Many names are used for Pacioli, such as Paciolo, Paciuolo, Fra Lucas de Borgo, etc.. The name Paciolo refers to a single person, whereas the name Pacioli is an abbreviation of "dei Pacioli" which means"one of the Paciolo's" (Taylor (1943)).
4. Pacioli is the first mathematician with an authentic portrait. Probably, the portrait was made by Jacobo de Barbari in 1495. He is also part of Piero della Francesca's painting the "Madonna and Child with Saints" (1472-74), which today remains in the Pinacoteca di Brera in Milan. Pacioli is the Franciscan monk behind the kneeling Federico da Montefeltro, Piero's most generous patron (Angelini (1985)).
5. For some reason, Pacioli preferred a portrait with the icosahexahedron (above one with a second Platonic solid). It might be that the construction of the icosahedron as it appears in the Divina Proportione was still unknown at the time of the painting.
6. See Volmer (1993) for an overview of Pacioli's work. Cantor ((1900), Kapitel 57) gives details on the Summa and the Divina proportione.
7. Pacioli follows the customs of the age and borrowed freely from various sources, often without giving the slightest credit (Smith ((1958), p252)).
8. The term "algebra" originates from the title hisāb "al-jabr" wa-l-muqābala (the calculus of reduction and cancellation) of the book by al-kuwarizm̄. The term "algorithm" is derived from its author's (name e.g. Hitti (1970) or Eves ((1969), 7.9). Our transliteration from Arabic follows Wehr (1976).
9. Multiplication and division were experienced as difficult operations. Despite the use of subtraction, Pacioli did not know negative numbers. Also, only the positive roots of an (quadratic) equation are searched for.
10. The solution of the cubic equation is due to Scipione del Ferro (ca.1465-1526).
11. The rule of false position already occurs in the Rhind Mathematical Papyrus (ca.1650 A.C.) (Robins and Shute (1987)).
12. The problem is stated in the present jargon. The development of probability theory dates from the 17th century.
13. This triangle can be easily solved by means of Heron's formula "$S^2 = l(l-a)\ (l-b)\ (l-c)$" which expresses the surface S of a triangle by means of the lengths a,b and c of the three sides [$l = 1/2(a+b+c)$] in combination with the equality "$S = l.r$" with r the radius of the inscribed circle. Although Pacioli was acquainted with both formulae, he preferred to elaborate a geometrical construction in order to convert the problem to an algebraical one.
14. The question on the "constructability" of numbers has led to the development of the Galois theory (Evariste Galois (1811 - 1832)).
15. Johann Kepler (1571-1630) and Louis Poinsot (1777-1859) discovered the four nonconvex (or starshaped) regular polyhedra. Building upon the work of Euler (1750), Cauchy (1811), and Lhuilier (1812-13), the German mathematician Bernhard Riemann (1826-1866) started the study of the topology of surfaces.
16. For a more detailed description, see e.g. de Waal (1927), Green (1930), Volmer (1993), and Hernandez-Esteve (1994).
17. Pacioli does not use complex entries, where more than one account is affected on the debit side and/or credit side.
18. The twelfth tractatus of the nineth distinctio (folios 211-224) provides tables on different currencies, measures, and weights.

19. A collection of more than 200,000 fragments, found in the Ben Ezra synagogue during its restoration in 1890. The Hebrew word "geniza", (in Arabic: *janāza,* i.e. burial) is derived from the Persian "ganj" which means "storehouse or treasure".
20. The original text is as follows: "A Luca Pacioli – che ebbero amico e consultore Leonardo da Vinci e Leon Battista Alberti – che primo die all'algebra linguaggio e struttura di scienza – avviò il gran trovato d'applicarla alla geometria – insegnò la scrittura doppia commerciale – dettò opere di matematica base e norma invariate alla postere lucubrazione – Il popula di S. Sepolcro – vergognando 370 anni di oblio – al gran concittadino – poneva – 1878".

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