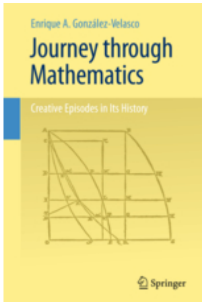
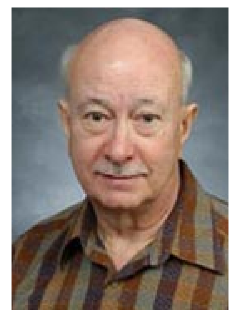
Journey through Mathematics. Creative Episodes in Its History, *Enrique A. González-Velasco* Springer, 2011 (xii+466 p.), hard cover, ISBN 978-0-387-92153-2, 59,95€ (net)



The book grew out of a mathematical history course given by the author. It has a list of 39 pages with references to historical publications which are amply cited and from which many parts are worked out in detail. This is organized in 6 richly illustrated chapters describing the evolution of concepts from ancient times till the XVIIIth century to what is now generally used in our calculus courses.

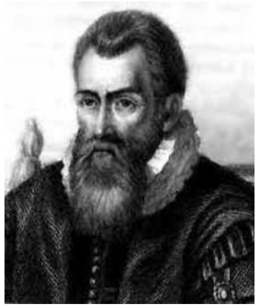
The first chapter on *trigonometry* starts with the Greek, the Indian, and the Islamic roots (mostly geometric) of trigonometric concepts. It all started with a stick fixed in the ground and shadows produced by the sun. The rectangular triangle was there and angles should be computed for astronomical computations. Fit the triangle in a circle and there is a bunch of geometric theorems to prove. Many were just helpful for computing the angles. Tables of chords were the predecessors of the goniometric tables. The diameter of the circle was divided into 120 parts and its circumference in 360 parts (implicitly setting $\pi \approx 3$) then $\sin\theta = \frac{\text{crd } 2\theta}{120}$ where crd 2θ is the length of the chord spanned by an angle 2θ. Thus one got approximate sine-tables although the idea is to work with lengths rather than with angles. One has to wait till the XVIth century for the real trigonometric tables to appear. They were used for computations in celestial mechanics in the work of François Viète (1540-1603). The cosine was not really needed and only appeared under its own name in 1620, while the notation sin. and cos. etc. (with dots), was only accepter as late as 1748.

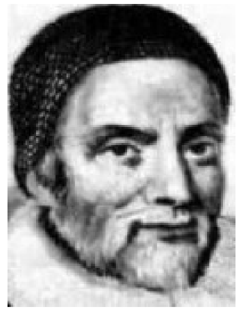

González-Velasco

The second chapter on the *logarithm* is a natural consequence of the trigonometric tables as an aid for computation. $\sin A \cdot \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ could be used to multiply numbers $x \approx \sin A$ and $y \approx \sin B$. John Napier (1550-1617) and Henri Briggs (1561-1630) worked together and came up with the concepts of logarithms in base e and in base 10 respectively. This was not a simple thing to do in days when limits nor decimal points existed. For example log(1) was not zero, so that it had to be taken into account in all the computations. Napier implicitly defined the basis e of his logarithm as a number whose logarithm was approximately 1: namely $1 - 10^{-k}$ to be precise. Briggs chose a basis 10 but also he used approximations $\log(10^7) = 0$ and $\log(10^6) = 10^{10}$. If we now define $\log(x) = \int_1^x (1/t)dt$, then this is thanks to Grégoire de Staint-Vincent (1584-1667) who connected logarithms to areas between the t-axis and the curve $1/t$: the hyperbolic logarithm. Later Isaac Newton (1642–1727) with his theory of 'infinite' sequences linked it with series expansions. Another century later Leonhard Euler (1707-1783) generalized it to a logarithms in an arbitrary basis.

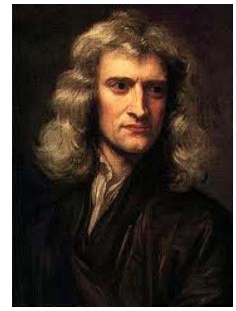
Complex numbers are introduced in chapter 3. This is tied up with the solution of a cubic equation as studied by Girolamo Cardano (1501-1576) and others. Polynomials were a big deal in those days and square roots of negative numbers were bound to appear. In 1545 he proposed to split 10 into two parts with product 40. Thus he solved $x(10-x) = 40$, giving the solutions $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$. Rafael Bombelli (1526-1572) described complex arithmetic. He called $+\sqrt{-1}$ *più di meno* and $-\sqrt{-1}$ *meno di meno* and gave the rules to multiply $\pm i$ with $\pm i$ as they are familiar to us now. By the way he also was the one who invented some kind of exponential notation. For example he denoted $x^3 = 15x + 4$ as $\overset{\overset{3}{\smile}}{1}\ a\ \overset{\overset{1}{\smile}}{15}\ p.\ \overset{\overset{0}{\smile}}{4}$. Euler even studied the logarithm of complex numbers, but it was only John Wallis (1616-1703) and Caspar Wessel (1745-1818) who gave the geometric interpretation and made the complex numbers accepted (if you can draw them, then they must exist). To William Rowan Hamilton (1805-1865) they were a couple of real numbers, giving their algebraic interpretation. Carl Friedrich Gauss (1777-1855) was the one to introduced the letter i for $\sqrt{-1}$ (although engineers like to use j instead).


Viète


Napier


Briggs

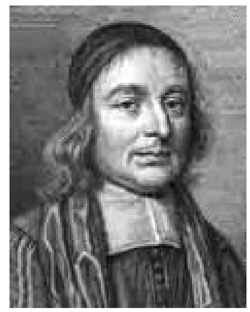

de St.Vincent


Newton


Euler


Cardano


Bombelli


Wallis


Wessel


Hamilton


Gauss


Leibniz


Gregory


Taylor


Maclaurin


de Fermat


Barrow


Fourier


Bolzano

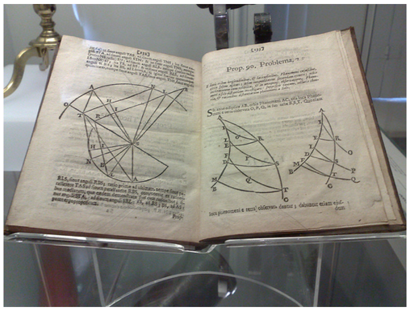

Cauchy


Dirichlet

da Cunha


D. Bernoulli

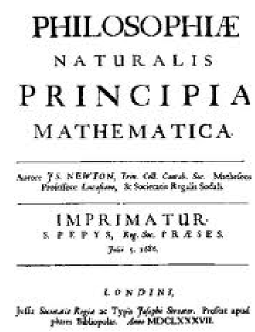

Abel



Gregory's book

Next chapter treats *infinite series*. Summation of (finite) numerical sequences, especially geometric sequences was known to the Egyptians and the Greek. But it was Gottfried Wilhelm Leibniz (1646-1716) who first summed the inverse of the triangular numbers $\frac{1}{n(n-1)}$ and Euler computed $\sum 1/k^2 = \pi^2/6$. As for function expansions, the Indians knew a series for $\sin(x)$ in the XIVth century, but in Europe one had to wait till the XVII-XVIIIth for Newton and Euler who developed a more general theory. However it was James Gregory (1638–1675) with his formulas for interpolating polynomials who later inspired Brook Taylor (1685-1731) and Colin Maclaurin (1698–1746) to develop their well known series.

Chapter 5 about *calculus* is the major part (about a quarter) of this book. Pierre de Fermat (1601-1665) contributed with his method of maxima and minima (derivatives) and his quadratures (integrals) of general hyperbolics $x^n y^m = 1$. So did Gregory and Isaac Barrow (1630-1677). Gregory wrote the first calculus book, but he, as well as Barrow did not talk about derivatives of integrals either. They studied tangents to a curve or a surface below a curve. Everything was still surprisingly inspired by geometry. Of course Newton and Leibniz are the main players here. Newton developed a theory of 'infinite' series (although there was always some 'last term'). Leibniz visited England and talked to Gregory but he basically developed his theory on his own. The problem seems to have been that his papers were very obscure and 'unreadable' for his fellow mathematicians. The well known dispute of plagiarism and who was the first to have invented 'calculus' was a consequence. González-Velasco makes a careful analysis of the work of both Newton and Leibniz and concludes that they worked independently.



Newton's book

The last chapter is about *convergence*. Even Leibniz's and Newton's ideas were still rather geometric: derivatives were still tangents and integrals were quadratures, thus essentially finite. The notion of limit was lacking, which was only developed later in contributions by Joseph Fourier (1768–1830), Bernard Bolzano (1781-1848), Augustin-Louis Cauchy (1789-1857), Peter Gustav Lejeune Dirichlet (1805-1859), and others. Fourier formulated Fourier series and it was Daniel Bernoulli (1700-1782) who raised the question if any function could be expanded into a Fourier series. The others worked out the different notions of convergence as we know it nowadays. Those are the names we usually connect with convergence of series. However there was a Portugese mathematician José Anastácio da Cunha (1744-1787) who is less known, but who was actually the first, although his much earlier contribution went largely unnoticed. González-Velasco devotes an extensive section to this and somehow helps to rehabilitate de Cunha's contribution.

da Cunha's book

González-Velasco has done a marvelous job by sketching this very readable historical tale. He stays as close as possible to the original way of thinking and the way of proving results. He is even using the notation and phrasing and explains how it would be experienced by scientists of those days. However at the same time he makes it quite understandable for us, readers, used to modern concepts and notation. A remarkable achievement that keeps you reading on and on.

In my opinion, this is not only compulsory reading for a course on the history of mathematics, but everyone teaching a calculus course should be aware of the roots and the wonderful achievements of the mathematical giants of the past centuries. They boldly went where nobody had gone before and paved the road for what we take for granted today.

Adhemar Bultheel