Between Square and Hexagon in Oresme's *Livre du Ciel et du Monde*

Lorenz Demey

Abstract. In logic, Aristotelian diagrams are almost always assumed to be closed under negation, and are thus highly symmetric in nature. In linguistics, by contrast, these diagrams are used to study lexicalization, which is notoriously not closed under negation, thus yielding more asymmetric diagrams. This paper studies the interplay between logical symmetry and linguistic asymmetry in Aristotelian diagrams. I discuss two major symmetric Aristotelian diagrams, viz. the square and the hexagon of opposition, and show how linguistic considerations yield various asymmetric versions of these diagrams. I then discuss a pentagon of opposition, which occupies an uneasy position between the square and the hexagon. Although this pentagon belongs neither to the symmetric realm of logic nor to the asymmetric realm of linguistics, it occurs several times in the literature. The oldest known occurrence can be found in the cosmological work of the 14th-century author Nicole Oresme.

Keywords. Square of opposition, JSB hexagon, pentagon of opposition, Aristotelian diagram, logical geometry.

There are more things in heaven and earth, Horatio,
Than are dreamt of in your philosophy.

— William Shakespeare, *Hamlet*

1 Introduction

Aristotelian diagrams visually represent the elements of some logical, lexical or conceptual field, and the relations of contradiction, contrariety, subcontrariety and subalternation holding between them. These diagrams are widely studied and used in philosophy and logic, but also in other disciplines that are concerned with logical reasoning, such as linguistics, mathematics and computer science. In particular, the disciplines of logic and linguistics offer highly complementary perspectives on Aristotelian diagrams. On the one hand, logicians almost always assume that the diagrams are closed under negation: if such a diagram contains φ, then it also contains $\neg\varphi$ (up to logical equivalence) (*Demey and Smessaert 2018b*, *Smessaert and Demey 2014*). Consequently, the Aristotelian diagrams studied in logic are highly regular and *symmetric* in nature.[1] On the other hand, linguists often focus on those concepts that are (primitively) lexicalized in natural language (e.g. English), i.e. concepts that can be expressed by means of a single word. The property of being lexicalized is notoriously not closed under negation: it is possible for a notion to be lexicalized, while its negation is not lexicalized (*Horn 1989*, *Seuren and Jaspers 2014*). Consequently, the Aristotelian diagrams studied in linguistics are often more *asymmetric* in nature.

The intricate interplay between logical symmetry and linguistic asymmetry has cropped up several times throughout the history of Aristotelian diagrams. My aim in this paper is to describe one particular manifestation of this dialectic, and to trace its historical roots. As such, the results of this paper not only constitute an interesting chapter in the historiography of logic, but they also provide valuable input for the contemporary systematic study of Aristotelian diagrams in *logical geometry* (*Demey 2018*, *2019a*, *forthcoming*; *Demey and Smessaert 2017*, *2018a*, *2018b*; *Smessaert and Demey 2014*, *2015*, *2017a*).

The paper is organized as follows. In Section 2, I discuss two frequently used symmetric Aristotelian diagrams, viz. the square and the hexagon of opposition, and I show how linguistic considerations yield various asymmetric versions of these diagrams. In Section 3, I then discuss a pentagon of opposition, and argue that it occupies an uneasy position in between the square and hexagon of opposition. Furthermore, I will show that although this pentagon belongs neither to the symmetric realm of logic nor to the asymmetric realm of linguistics, several occurrences of it can be found across the literature. In Section 4, we then turn to what is (to the best of my knowledge) the oldest occurrence of the pentagon of opposition. Quite surprisingly, this pentagon is found not in a logical or linguistic treatise, but rather in the cosmological work of the 14th-century philosopher Nicole Oresme. Finally, Section 5 wraps things up, and offers some concluding remarks.

2 From the Square to the Hexagon

Without a doubt, the oldest and most well-known Aristotelian diagram is the *square of opposition* (*Parsons 2017*). Figure 1 shows squares of opposition for logical systems such as syllogistics, modal logic, and propositional logic. I will make use of the well-established vowel code (A/E/I/O) for labeling the four corners of the square, as shown in Figure 2(a). This is a typical example of an Aristotelian diagram as studied in logic, with a high degree of symmetry. It is closed under negation: the negation of I is E (and vice versa), and the negation of A is O (and vice versa).

[1] This symmetry can mathematically be described in various ways, using tools from Euclidean geometry, group theory and graph theory (*Demey and Smessaert 2014*, *2016b*, *2017*, *2018a*; *Smessaert and Demey 2016*).

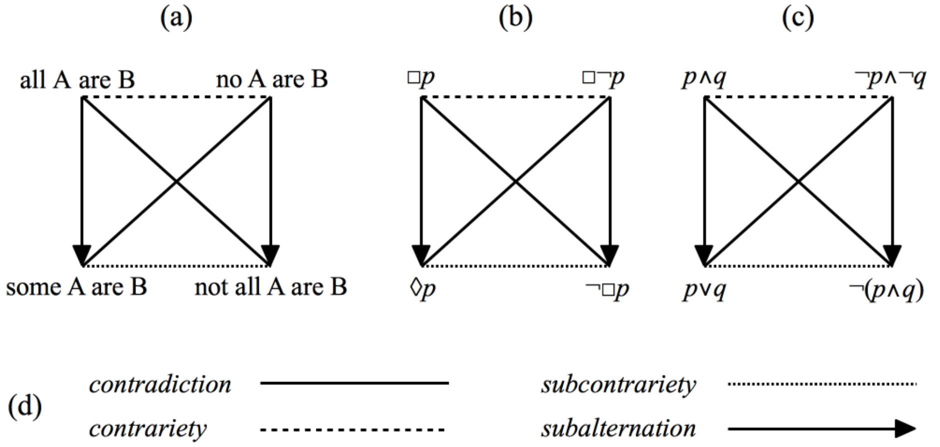

Figure 1: Squares of opposition for (a) syllogistics, (b) modal logic, (c) propositional logic; (d) code for visualizing the Aristotelian relations.

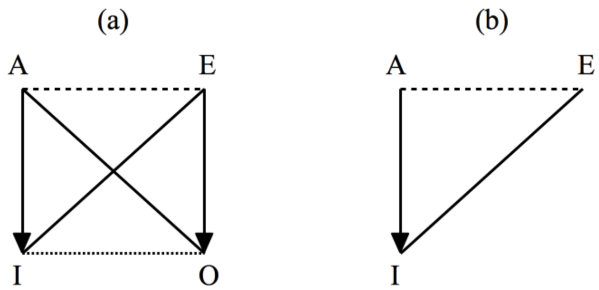

Figure 2: (a) Abstract square of opposition; (b) the lexicalized part of the square.

From a linguistic perspective, there is a significant difference between the A-, I- and E-corners of the square on the one hand, and the O-corner on the other. Typically, A, I and E are lexicalized in natural language. For example, with the quantifiers we have *all*, *some* and *no*; with the modalities we have *necessary*, *possible* and *impossible*; and with the propositional connectives we have *and*, *or* and *nor*. By contrast, O is not lexicalized: there do not exist words such as **nall*, **nnecessary* and **nand*. This is known as the problem of the non-lexicalization of the O-corner (*Horn 1989*, *2012*; *Katzir and Singh 2013*). In terms of diagrams, it means that we disregard the O-corner, and focus on the A-, I- and E-corners. This yields a *triangle of opposition*, as shown in Figure 2(b); also cf. *Horn 1989* (p. 253). Note that this triangle is not closed under negation: it contains A, but it does not contain the negation of A (viz. O).

Let us now switch back to the logical perspective. The square of opposition is closed under the Boolean operation of negation. Hence, a natural question to ask is whether it is also closed under the other Boolean operations of conjunction and disjunction.[2] This is not the case: (i) the square contains I and O, but it does not contain (a proposition that is logically equivalent to) their conjunction I $\wedge$ O (which is often labeled 'Y'); similarly, (ii) the square contains A and E, but it does not contain (a proposition that is logically equivalent to) their disjunction A $\vee$ E (which is often labeled 'U'). It can be shown that these are the only two contingent Boolean combinations that are missing from the square. We can add them to the square, thereby obtaining a *hexagon of opposition*, as shown in Figure 3(a). This hexagon is the Boolean closure of the square: it is the smallest Aristotelian diagram that (i) is closed under all Boolean operations and (ii) contains the square as a subdiagram. It was first studied in the 1950s by *Jacoby 1950*, *Sesmat 1951* and *Blanché 1953*, and is therefore nowadays called the 'Jacoby-Sesmat-Blanché (JSB) hexagon'.[3] The broader historical background to this crucial logical development is described in *Jaspers and Seuren 2016*.

Just like the square of opposition, the JSB hexagon is nowadays used so frequently that it is unrealistic to attempt to enumerate all its applications. However, it should be emphasized that different authors use widely different diagrams to visualize this constellation of six formulas. By far the most common visualization is based on a regular hexagon with an equilateral triangle of contraries (A – E – Y), as shown in Figure 3(a). However, *Wybraniec-Skardowska 2016* systematically visualizes this constellation by means of a regular hexagon with an isosceles triangle of contraries, as shown in Figure 3(b). Jacoby chooses to draw the JSB hexagon not as an hexagon at all, but rather makes use of the self-duality of regular triangles, and draws it as a small triangle of contraries inside a large triangle of subcontraries, as shown in Figure 3(c) (*Jacoby 1950*, p. 46; *1960*, p. 144). Making use of the same self-duality principle, *Sesmat 1951* (p. 450), *Horn 2012* (p. 400) and *Dubois and Prade 2012* (p. 155) draw it as a small triangle of subcontraries inside a large triangle of contraries, as shown in Figure 3(d).[4] Finally, *Smessaert 2009* (p. 311ff.) and *García-Cruz 2017* (p. 259ff.) make use of yet another visualization technique: they move from two- to three-dimensional space, and draw the JSB 'hexagon' as a regular octahedron, as shown in Figure 3(e). All these different diagrams exhibit exactly the same information (the same formulas and the same Aristotelian relations). *Demey and Smessaert 2016b* show how such informationally equivalent visualizations can be fruitfully studied based on their number of geometrical symmetries. In the remainder of this paper, we will disregard this variety in visualizations, and work with the 'ordinary' visualization of the JSB hexagon, as shown in Figure 3(a).

[2] Given the interdefinability of conjunction and disjunction (in the presence of negation), it follows that (i) if a diagram is closed under negation and conjunction, it is closed under disjunction as well, and (ii) if a diagram is closed under negation and disjunction, it is closed under conjunction as well.

[3] This type of hexagon has also been used, in the secondary literature, to analyze certain philosophical theories by historical authors such as Marsilius of Padua (*Tierney 2007*), Gottfried Achenwall (*Hruschka 1986*) and Immanuel Kant (*Joerden 1995*). However, as far as we know, none of these historical authors made use of any actual hexagonal diagrams themselves.

[4] For reasons of visual simplicity, the subalternation relations have been left out of the two 'nested triangle' visualizations in Figure 3(c–d).

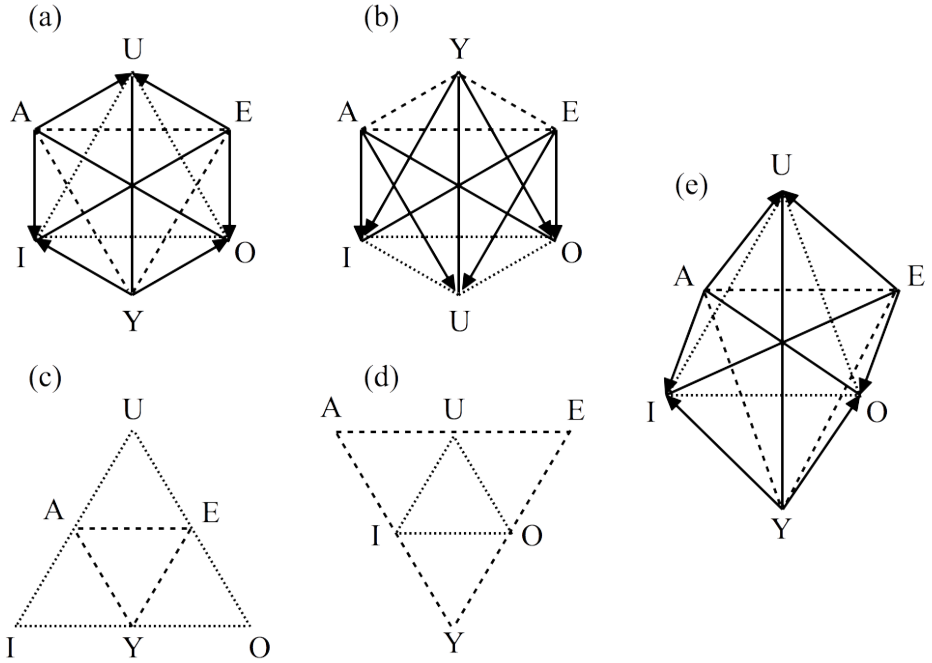

Figure 3: Various visualizations of the abstract JSB hexagon: (a) regular hexagon with equilateral contrariety triangle, (b) regular hexagon with isosceles contrariety triangle, (c) small contrariety triangle inside large subcontrariety triangle, (d) small subcontrariety triangle inside large contrariety triangle, (e) octahedron.

From a linguistic perspective, there is again a significant difference between the Y- and the U-corner of the JSB hexagon. Typically, Y is lexicalized in natural language (often in the same way as I). For example, with the quantifiers we have (bilateral) *some*; with the modalities we have (bilateral) *possible*; and with the propositional connectives we have (exclusive) *or*. By contrast, U is not lexicalized: there do not exist words such as **allorno*, **necessaryorimpossible* and **andornor*. Ideally, we would want to have a theory that can simultaneously explain the non-lexicalization of the O-corner and that of the U-corner. *Seuren and Jaspers 2014* have developed precisely such a theory, based on a recursive partitioning process of logical space. In terms of diagrams, the linguistic perspective means that we disregard the O- and U-corners, and focus on the A-, I-, E- and Y-corners. This yields a *kite*, as shown in Figure 4(b). Note, again, that this kite is not closed under negation: it contains A and Y, but it does not contain the negation of A (viz. O) or the negation of Y (viz. U). *Roelandt 2016* makes extensive use of kite diagrams in his analysis of the semantics of binary and gradable adjectives. Furthermore, the kite was already discussed by *Blanché 1966* (pp. 93–94), who called it 'an irregular tetrad' (*une tétrade irrégulière*).

We should here also mention the so-called 'Jespersen triangle', another dia-

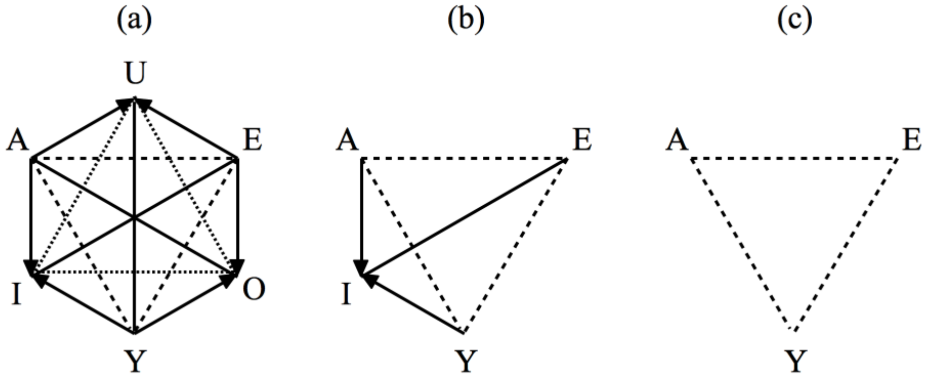

Figure 4: (a) Abstract JSB hexagon; (b) the lexicalized part of the JSB hexagon; (c) Jespersen triangle.

gram that is linguistically motivated, and thus asymmetric (i.e. not closed under negation); cf. Figure 4(c). In comparison with the kite, the Jespersen triangle drops the I-corner as well (since it is typically co-lexicalized with the Y-corner), and thus only keeps A, E and Y (i.e. exactly the three contraries from the original JSB hexagon). This diagram is named after the linguist *Jespersen 1917*, *1924*, and has also been used by other linguists, such as *Geerts and Melis 1976* (p. 111) and *Horn 1989* (pp. 218, 252), *2012* (p. 398). However, it has also been used in several other contexts. For example, *Jacoby 1950* (p. 38) briefly discusses it before moving on to the full, symmetric (i.e. closed under negation) JSB hexagon. It has also been used in the philosophy of law, by *Hruschka and Joerden 1987* (p. 108) and *Halpin 2003* (pp. 45, 49ff.). In recent years it has even found applications in disciplines such as psychology (*Schmidt and Thompson 2008*, p. 219) and theology (*Boyd 2010*, p. 53; *Demey 2019*, p. 390).

3 Between Square and Hexagon

Unfortunately, logical and intellectual developments hardly ever proceed in the clear-cut and smooth fashion that I have described above. For example, one might want to add the Y-corner to the square (because it is lexicalized and/or represents an interesting philosophical notion), *without* having to add the U-corner as well (because that one is not lexicalized and/or does not represent an interesting philosophical notion). In this way one obtains a *pentagon of opposition*, as shown in Figure 5(a). This pentagon occupies an uneasy position 'between' the square of opposition and the JSB hexagon. On the one hand, it does not fit well with the logical perspective (because it is not closed under negation: it contains Y, but not the negation of Y), and on the other hand, it does not fit well with the linguistic perspective either (because it contains a notion that is typically not lexicalized, viz. O). Diagrammatically speaking, the pentagon can be seen as the result of superimposing the square of opposition and the kite: the former is a 'logic-oriented', symmetric diagram, whereas the latter is a 'linguistics-oriented', asymmetric diagram. Alternatively, the pentagon can also be seen as the result of superimposing the (logic-oriented) square of opposition and the (linguistics-oriented) Jespersen triangle.

Unnatural though it may be, concrete instances of the pentagon can effectively be found in the extant literature.[5] For example, in 1952, Blanché already provided a table of the five propositions that make up the pentagon, and noted that his 'five-lined table may be contracted into a four-lined table by omitting Y [thus keeping A, E, I and O, i.e. the square of opposition from Figure 2(a)], or into a three-lined one by omitting I and O [thus keeping A, E and Y, i.e. the Jespersen triangle from Figure 4(c)]' (*Blanché 1952*, p. 375). He also mentions the possibility of contracting his table to the 'four-lined table A, E, I, Y', i.e. the kite from Figure 4(b), but dismisses this as 'irregular and somewhat unnatural' (*Blanché 1952*, p. 375) — thereby showing himself to be first and foremost a logician, rather than a linguist. Despite his detailed discussion of the five propositions (A, I, E, O, Y), Blanché did not draw the actual pentagon for them. (One year later, he added the sixth proposition (U) to his table, and then did draw a JSB hexagon based on these six propositions; cf. *Blanché 1953*.)

Kalinowski 1953 (p. 162) made use of an actual pentagon of opposition to illustrate his account of the normative propositions; see Figure 5(b).[6] In this pentagon, A represents that a given action α is *obligatory*, while E represents that α is forbidden. Correspondingly, I represents that it is *permitted* to do α and O represents that it is *permitted not* to do α. The Y-corner represents that α is *gratuitous*: it is permitted to do α and it is permitted not to do α. This deontic version of the pentagon has also been studied by *Moore 1998* (p. 114). In 1972, Kalinowski returned to this topic, and although he repeats his pentagon, he also provides the 'full' JSB hexagon (which also includes the U-corner, representing that α is either obligatory or forbidden), and emphasizes that he considers the latter to be the superior Aristotelian diagram (*Kalinowski 1972*, p. 119).

Next, *Gildin 1970* (p. 102) uses a pentagon of opposition to analyze Aristotle's moral theory; see Figure 5(c).[7] In this pentagon, A represents the vice of *rash-*

[5]Apart from the ones discussed in this section, pentagons can also be found in *Menne 1954* (p. 75), *Kneale and Kneale 1962* (p. 86), *Gasser 1987* (p. 26) and *Choudhury and Chakraborty 2016* (p. 229).

[6]Kalinowski did not draw a pentagon as such. Rather, he moved from two- to three-dimensional space, and drew a pyramid, with A, I, E and O at the base, and Y at the top. This pyramid diagram is informationally equivalent to the pentagon diagram shown in Figure 5(b). The relationship between the (3D) pyramid and the (2D) pentagon is entirely analogous to the relationship between the (3D) octahedron and the (2D) regular JSB hexagon in Figure 3(a/e).

[7]Interestingly, Gildin himself called his diagram a 'square of opposition' rather than a pentagon — perhaps because of the popularity of the square, in contrast to the obscurity of the pentagon. Nevertheless, the diagram does contain a Y-corner, and should thus be seen as being first and foremost a *pentagon*.

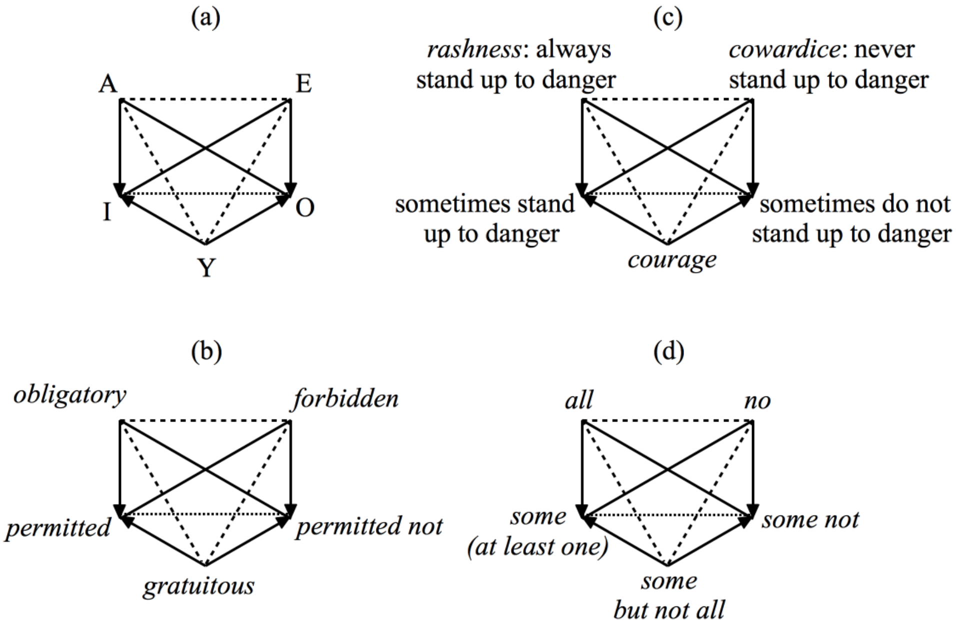

Figure 5: (a) Abstract pentagon of opposition; (b) Kalinowski's deontic pentagon; (c) Gildin's moral pentagon; (d) Horn's quantificational pentagon.

ness (always standing up to danger) and E represents the vice of *cowardice* (never standing up to danger). Correspondingly, I stands for sometimes standing up to danger, while O stands for sometimes not standing up to danger. However, the true virtue, *courage*, occupies a 'middle ground' between the two vices, and is thus represented by the Y-corner: sometimes standing up to danger and sometimes not standing up to danger.

Finally, *Horn 2012* (p. 403) uses a pentagon while explaining his neo-Gricean account of the non-lexicalization of the O-corner; see Figure 5(d). In this pentagon, A stands for *all* and E stands for *no*. The O-corner is not lexicalized, and stands for *some not*. The I- and Y-corners have the same lexicalization: I stands for unilateral *some* (i.e. *at least one*), while Y stands for bilateral *some* (i.e. *some but not all*). Note that if $\langle all, some \rangle$ is taken to be a Horn scale, then the Y-corner can be viewed as the result of combining the semantics of the I-corner (*at least one*) with its scalar implicature (*not all*).

4 Oresme's *Livre du Ciel et du Monde*

The concrete examples of pentagons that have just been discussed, are all fairly recent (20th century). In this section, however, I will discuss what is — to the best of my knowledge — the oldest example of a pentagon in the literature. This example is due to the 14th-century author Nicole Oresme, one of the most eminent scholastic philosophers, mathematicians and scientists (*Celeyrette and Grellard 2014*, *Kirschner 2017*).[8]

Oresme lived in France from around 1320 to 1382 (*Burton 2007*). His early career was spent at the University of Paris (*Courtenay 2000*). From 1362 until his death, he served Charles, the dauphin of France, who was crowned King Charles V in 1364. Oresme was tasked by Charles to produce French translations of, and commentaries on, several of Aristotle's works (*Kirschner 2017*).[9] One of these works was Aristotle's cosmological treatise *On the Heavens* (Περὶ οὐρανοῦ). This treatise was translated from Greek into Latin (as *De Caelo et Mundo*), first by Gerard of Cremona (1170), and later by Robert of Lincoln and William of Moerbeke (1250–1265).[10] Oresme used the latter as the basis for his translation from Latin into French (as *Le livre du ciel et du monde*). The first printed version of Oresme's translation appeared in the journal *Mediaeval Studies* in the early 1940s (*Menut and Denomy 1941*, *1942*, *1943*); a revised version, in which the Middle French text is accompanied by a contemporary English translation, was published a few decades later (*Menut and Denomy 1968*).

In his *Livre du ciel et du monde*, which has been called 'perhaps Oresme's greatest scientific treatise' (*Grant 1978*, p. 107), Oresme discusses Aristotle's remarks regarding objects that have/do not have a beginning and objects that have/do not have an end, and he illustrates his discussion by means of a pentagon. He explicitly recognizes the similarity between his diagram and the more common square of opposition (*Menut and Denomy 1968*, pp. 220–221):

> In order to illustrate this, I clarify it by means of a figure very similar to that used to introduce children to logic.
>
> (*Et pour ce miex entendre, je le desclaire en une figure presque semblable a une que l'en fait pour la premiere introducion des enfans en logique.*)

It is perhaps worth pointing out that one of Oresme's colleagues at the University of Paris was John Buridan.[11] Buridan, who has been called 'one of the best

[8]I have recently learned that there also appear pentagons of opposition in the *Moralis seu Politica Logica* (1680) by Juan Caramuel y Lobkowitz (1606–1682) (*Dvořák 2008*, p. 652). However, Caramuel's pentagons are much more recent than those of Oresme and, most importantly, they contain several mistakes. In particular, the relations from the Y-vertex to the A- and E-vertices are contrarieties — cf. Figure 5(a) —, but Caramuel labeled them contradictories.

[9]From a historical-linguistic perspective, it is interesting to note that through these translations, Oresme had a considerable influence on the development of scientific and philosophical vocabulary in medieval French (*Kirschner 2017*, *Menut and Denomy 1968*).

[10]The Dominican William of Moerbeke (1215–1286) was one of the most prolific translators of philosophical and scientific treatises from Greek into Latin. He maintained a vivid correspondence with many prominent authors of his time, such as his fellow Dominican, Thomas Aquinas (1225–1274) (*Brams and Vanhamel 1989*). As is suggested by his name, William originates from the Flemish town of Moerbeke, near Geraardsbergen.

[11]The exact nature of the relation between Buridan and Oresme is not entirely clear. Early scholars such as *Duhem 1958* (p. 216) and later also *Clagett 1974* (p. 223), *Grant 1978* (p. 106) and *Patar*

logicians of all times' (*Dutilh Novaes 2014*, p. 610), famously made use of several *octagons* of opposition in order to explain and illustrate his logical theorizing (*Klima 2001*, *Read 2012*, *Demey 2019a*). These facts shed new light on Oresme's otherwise surprising choice to make use of a pentagon of opposition. After all, because of his thorough familiarity with Buridan's work, the idea of extending the usual square of opposition into a larger, more complex diagram might have come more naturally to Oresme than it would have to many of his contemporaries.[12]

A contemporary version of Oresme's pentagon is shown in Figure 6(a). The A-corner stands for 'always possible to be' (*tousjours possible estre*), the E-corner stands for 'always possible not to be' (*tousjours possible non estre*), the I-corner stands for 'not always possible not to be' (*non pas tousjours possible non estre*) and the O-corner stands for 'not always possible to be' (*non pas tousjours possible estre*).[13] Finally, the Y-corner is simply labeled 'the intermediate' (*le moien*), but from the ensuing text it is clear that this was taken to mean I $\wedge$ O. Oresme used this pentagon to clarify the following passage from (William of Moerbeke's Latin translation of) Aristotle (*Menut and Denomy 1968*, pp. 220–221, my emphases):[14]

> Therefore, it is necessary that the two negations of the two [contraries][15], —that is, the two subcontraries— be said of the same identical thing

1995 (p. 12) held that Oresme was directly a student under Buridan. More recently, scholars such as *Courtenay 2000* (p. 548) and *Thijssen 2004* (pp. 23–27) have convincingly argued that Oresme cannot have studied directly under Buridan, because they belonged to different 'nations' of the Faculty of Arts at the University of Paris (Oresme belonged to the Norman nation, Buridan to the Picard nation). The received view now seems to be that Buridan and Oresme are best seen as members of 'a small intellectual network of nearly contemporary masters of arts, who were familiar with each other's work and at times responded to one another' (*Thijssen 2004*, p. 42); also see *Burton 2007* (p. 33) and *Courtenay 2004* (p. 7). This mutual familiarity is clear from the fact that Buridan explicitly refers in his writings to 'reverendus magister Nycolaus Oresme' (*Burton 2007*, p. 33). In one of his later works, Oresme also recounts how he remembers that a certain magister had died; this unspecified magister is widely believed to have been Buridan (*Hansen 1985*, pp. 48–49).

[12]Indeed, apart from Buridan and Oresme, nearly all medieval logicians seem to have restricted themselves to (various versions of) the square of opposition. One possible other exception in this regard is the 13th-century author William of Sherwood (*Kretzmann 1966*, *Khomskii 2012*).

[13]Note that by themselves, these A-, E-, I- and O-corners not only constitute an *Aristotelian square* (with relations of contradiction, (sub)contrariety and subalternation), but also a *duality square* (with relations of internal negation, external negation and duality) (*Demey and Smessaert 2016a*, *Smessaert and Demey 2017b*). Furthermore, since the operator generating this square is itself the result of composing a temporal operator (*tousjours*) with a modal operator (*possible*), the square can naturally be extended to a duality *cube* (*Demey 2012*).

[14]The idea that there is something intermediate between 'always possible to be' and 'always possible not to be' can thus be traced back to Aristotle himself. Oresme's own innovation was that he saw that this intermediate position could best be understood by extending the usual square of opposition to a pentagon (recall the previous quotation: 'In order to illustrate this. . . '). Thanks to an anonymous reviewer for some helpful discussion about this point.

[15]In the English translation by *Menut and Denomy 1968* (p. 221) we read 'contradictories' at this place, but this is clearly incorrect. In most manuscripts, the original French text simply has 'con', which is erroneously conjectured to stand for 'con⟨tradictoires⟩' (*Menut and Denomy 1968*, p. 220). Furthermore, one of the remaining manuscripts of Oresme's *Livre* explicitly has 'contraires' here, which is as it should be (*Menut and Denomy 1968*, p. 220, Footnote 2).

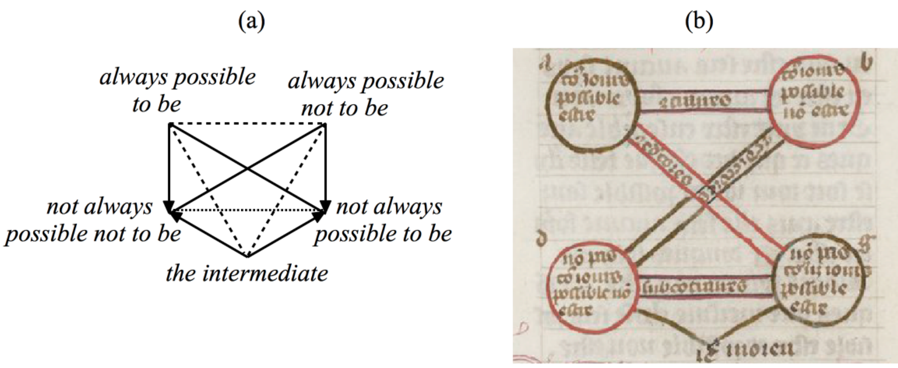

Figure 6: (a) Oresme's pentagon of opposition; (b) the actual pentagon in BnF Ms. franç. 1082, fol. 51r.

> and that this thing should be intermediate between always being and always not being. It is what is capable of being and of not being, for *each of the two subcontraries will sometimes be true*, but not always so.
>
> (*Et pour ce convient par necessité que les negacions des .ii. [contraires], c'est assavoir les .ii. subcontraires, soient dictes d'une meisme chose et que celle chose soit moienne entre touzjours estre et touzjours non-estre. Et est la chose qui est possible estre et possible non-estre, quar chascune des .ii. negacions, qui sont subcontraires, sera vraie aucune foys, pousé que ce ne soit pas touzjours.*)

We conclude this section by taking a look at Oresme's actual pentagon, as it appears in one of the six remaining manuscripts of the *Livre du ciel et du monde*, viz. Ms. franç. 1082 of the Bibliothèque nationale de France (BnF), which is freely available online in digital format (*Oresme 2019*), and which constitutes the main source for the edition and translation by *Menut and Denomy 1941*, *1942*, *1943*, *1968*. On fol. 51r of this manuscript, we find the diagram that is reproduced here as Figure 6(b). Note that the diagram is not complete; for example, the subalternations from A to I and from E to O are missing, as well as the contrarieties between A and Y and between E and Y. Nevertheless, the key aspect of the pentagon, viz. the addition of a fifth corner (Y, *le moien*), is clearly discernible, thus making Oresme the earliest known author to have used this unusual type of Aristotelian diagram.

5 Conclusion

In this paper I have examined a strange Aristotelian diagram, viz. the pentagon of opposition. This pentagon occupies an uneasy position between the more common square of opposition and JSB hexagon. It also transcends the boundaries between the logical perspective on Aristotelian diagrams (with its focus on closure under negation and the resulting symmetry) and the linguistic perspective on those diagrams (with its focus on lexicalization and the resulting asymmetry).

Despite its peculiar features, the pentagon has nevertheless been used several times across the literature. This observation is highly valuable for *logical geometry*, i.e. the contemporary, systematic study of Aristotelian diagrams. For example, logical geometry typically makes the simplifying logical assumption that Aristotelian diagrams are closed under negation (*Demey and Smessaert 2016c*, *2018b*; *Smessaert and Demey 2014*, *2017b*), together with the simplifying geometrical assumption that the diagrams are highly regular, symmetric polygons/polyhedra (*Demey and Smessaert 2016b*, *2017*, *2018a*). It was already known that there exist Aristotelian diagrams which do not satisfy these simplifying assumptions, but these were treated as exceptions, and dealt with in an ad hoc fashion. However, the findings of the current paper show that there also exist Aristotelian diagrams which do not satisfy the simplifying assumptions, yet cannot be brushed off so easily. If logical geometry purports to be a truly comprehensive and systematic study of Aristotelian diagrams, then it will also need to analyze the pentagon of opposition in more detail.

Finally, I have also discussed the earliest known occurrence of the pentagon, which can be found in Nicole Oresme's late-14th-century *Livre du ciel et du monde*. The fact that an odd diagram such as the pentagon is found, not in an ordinary philosophy textbook but rather in a treatise on heaven and earth, clearly illustrates the truth of Hamlet's reply to Horatio.

Acknowledgments

Thanks to Jens Lemanski, Hans Smessaert, Margaux Smets and an anonymous reviewer for their useful feedback. An earlier, much shorter version of this paper appeared in an online Festschrift in honor of Dany Jaspers's 60th birthday (http://www.dj60.be).

Funding

The author holds a Postdoctoral Fellowship of the Research Foundation–Flanders (FWO).

References

Blanché, R. 1952. 'Quantity, modality, and other kindred systems of categories', *Mind*, **61**: 369–375.

Blanché, R. 1953. 'Sur l'opposition des concepts', *Theoria*, **19**: 89–130.

Blanché, R. 1966. *Structures Intellectuelles*, Paris: Vrin.

Boyd, G. A. 2010. 'Two ancient (and modern) motivations for ascribing exhaustively definite foreknowledge to God: a historic overview and critical assessment', *Religious Studies*, **46**: 41–59.

Brams, J. and Vanhamel, W., eds. 1989. *Guillaume de Moerbeke: recueil d'études à l'occasion du 700e anniversaire de sa mort (1286)*, Leuven: Leuven University Press.

Burton, D. 2007. *Nicole Oresme's De visione stellarum (On seeing the stars). A Critical Edition of Oresme's Treatise on Optics and Atmospheric Refraction, with an Introduction, Commentary, and English Translation*, Leiden: Brill.

Celeyrette, J. and Grellard, C., eds. 2014. *Nicole Oresme philosophe. Philosophie de la nature et philosophie de la connaissance à Paris au XIVe siècle*, Turnhout: Brepols.

Choudhury, L. and Chakraborty, M. K. 2016. 'Singular propositions, negation and the square of opposition', *Logica Universalis*, **10**: 215–231.

Clagett, M. 1974. 'Oresme, Nicole', in: C. C. Gillispie, ed., *Dictionary of Scientific Biography, Volume 10*, New York, NY: Charles Scribner's Sons, 223–230.

Courtenay, W. 2000. 'The early career of Nicole Oresme', *Isis*, **91**: 542–548.

Courtenay, W. 2004. 'The university of Paris at the time of Jean Buridan and Nicole Oresme', *Vivarium*, **42**: 3–17.

Demey, L. 2012. 'Algebraic aspects of duality diagrams', in: P. T. Cox, B. Plimmer, and P. Rodgers, eds., *Diagrams 2012*, LNCS 7352, Berlin: Springer, 300–302.

Demey, L. 2018. 'Computing the maximal Boolean complexity of families of Aristotelian diagrams', *Journal of Logic and Computation*, **28**: 1323–1339.

Demey, L. 2019a. 'Boolean considerations on John Buridan's octagons of oppositions', *History and Philosophy of Logic*, **40**: 116–134.

Demey, L. 2019b. 'A hexagon of opposition for the theism/atheism debate', *Philosophia*, **47**: 387–394.

Demey, L. Forthcoming. 'Aristotelian diagrams for semantic and syntactic consequence', *Synthese*, DOI 10.1007/s11229-018-01994-w.

Demey, L. and Smessaert, H. 2014. 'The relationship between Aristotelian and Hasse diagrams', in: T. Dwyer, H. Purchase, and A. Delaney, eds., *Diagrams 2014*, LNCS 8578, Berlin: Springer, 213–227.

Demey, L. and Smessaert, H. 2016a. 'Duality in logic and language', in: J. Fieser and B. Dowden, eds., *Internet Encyclopedia of Philosophy*, Martin, TN: University of Tennessee.

Demey, L. and Smessaert, H. 2016b. ‘The interaction between logic and geometry in Aristotelian diagrams’, in: M. Jamnik, Y. Uesaka, and S. E. Schwartz, eds., *Diagrams 2016*, LNCS 9781, Berlin: Springer, 67–82.

Demey, L. and Smessaert, H. 2016c. ‘Metalogical decorations of logical diagrams’, *Logica Universalis*, **10**: 233–292.

Demey, L. and Smessaert, H. 2017. ‘Logical and geometrical distance in polyhedral Aristotelian diagrams in knowledge representation’, *Symmetry*, **9**(10): 204.

Demey, L. and Smessaert, H. 2018a. ‘Geometric and cognitive differences between Aristotelian diagrams for the Boolean algebra $\mathbb{B}_4$’, *Annals of Mathematics and Artificial Intelligence*, **83**: 185–208.

Demey, L. and Smessaert, H. 2018b. ‘Combinatorial bitstring semantics for arbitrary logical fragments’, *Journal of Philosophical Logic*, **47**: 325–363.

Dubois, D. and Prade, H. 2012. ‘From Blanché’s hexagonal organization of concepts to formal concept analysis and possibility theory’, *Logica Universalis*, **6**: 149–169.

Duhem, P. 1958. *Le système du monde. Histoire des doctrines cosmologiques de Platon à Copernic. Tome VIII*, Paris: Hermann.

Dutilh Novaes, C. 2014. ‘Review of: Johannes Buridanus, Summulae de Locis Dialecticis’, *Journal of the History of Philosophy*, **52**: 609–610.

Dvořák, P. 2008. ‘Relational logic of Juan Caramuel’, in: D M. Gabbay and J. Woods, eds., *Handbook of the History of Logic. Volume 2: Mediaeval and Renaissance Logic*, Amsterdam: North-Holland, 645–665.

García-Cruz, J. D. 2017. ‘From the square to octahedra’, in: J.-Y. Béziau and G. Basti, eds., *The Square of Opposition: A Cornerstone of Thought*, Berlin: Springer, 253–272.

Gasser, J. 1987. *La syllogistique d’Aristote à nos jours*, Neuchâtel: Centre de recherches sémiologiques – Université de Neuchâtel.

Geerts, W. and Melis, L. 1976. ‘Remarques sur le traitement des modalités en linguistique’, *Langages*, **43**: 108–115.

Gildin, H. 1970. ‘Aristotle and the moral square of opposition’, *The Monist*, **54**: 100–105.

Grant, E. 1978. ‘Scientific thought in fourteenth-century Paris: Jean Buridan and Nicole Oresme’, *Annals of the New York Academy of Sciences*, **317**: 105–124.

Halpin, A. 2003. 'Fundamental legal conceptions reconsidered', *Canadian Journal of Law and Jurisprudence*, **16**: 41–54.

Hansen, B. 1985. *Nicole Oresme and the Marvels of Nature: A Study of His De causis mirabilium*, Toronto: Pontifical Institute of Mediaeval Studies.

Horn, L. R. 1989. *A Natural History of Negation*, Chicago, IL: University of Chicago Press.

Horn, L. R. 2012. 'Histoire d'*O: Lexical pragmatics and the geometry of opposition', in: J.-Y. Béziau and G. Payette, eds., *The Square of Opposition. A General Framework for Cognition*, Bern: Peter Lang, 393–426.

Hruschka, J. 1986. *Das deontologische Sechseck bei Gottfried Achenwall im Jahre 1767*, Göttingen: Joachim Jungius-Gesellschaft der Wissenschaften.

Hruschka, J. and Joerden, J. 1987. 'Supererogation: Von deontologischen Sechseck zum deontologischen Zehneck', *Archiv für Rechts- und Sozialphilosophie*, **73**: 93–123.

Jacoby, P. 1950. 'A triangle of opposites for types of propositions in Aristotelian logic', *New Scholasticism*, **24**: 32–56.

Jacoby, P. 1960. 'Contrariety and the triangle of opposites in valid inferences', *New Scholasticism*, **34**: 141–169.

Jaspers, D. and Seuren, P. 2016. 'The square of opposition in Catholic hands: A chapter in the history of 20th-century logic', *Logique et Analyse*, **59** (233): 1–35.

Jespersen, O. 1917. *Negation in English and Other Languages*, Copenhagen: A.F. Høst.

Jespersen, O. 1924. *Philosophy of Grammar*, London: Allen & Unwin.

Joerden, J. 1995. 'Überlegungen zum Begriff des Unrechtsstaats. Zugleich eine Annäherung an eine Passage zur Staatstypologie in Kants Anthropologie', *Jahrbuch für Recht und Ethik*, **3**: 253–265.

Kalinowski, G. 1972. *La Logique des Normes*, Paris: Presses Universitaires de France.

Kalinowski, J./G. 1953. 'Teoria zdań normatywnych / Théorie des propositions normatives', *Studia Logica*, **1**: 113–184.

Katzir, R. and Singh, R. 2013. 'Constraints on the lexicalization of logical operators', *Linguistics and Philosophy*, **36**: 1–29.

Khomskii, Y. 2012. 'William of Sherwood, singular propositions and the hexagon of opposition', in: J.-Y. Béziau and G. Payette, eds., *The Square of Opposition. A General Framework for Cognition*, Bern: Peter Lang, 43–60.

Kirschner, S. 2017. ‘Nicole Oresme’, in: E. N. Zalta, ed., *Stanford Encyclopedia of Philosophy (Summer 2017 Edition)*, Stanford, CA: CSLI.

Klima, G, ed. 2001. *John Buridan, Summulae de Dialectica*, New Haven, CT: Yale University Press.

Kneale, W. and Kneale, M. 1962. *The Development of Logic*, Oxford: Clarendon Press.

Kretzmann, N. 1966. *William of Sherwood’s Introduction to Logic*, Westport, CT: Greenwood Press Publishers.

Menne, A. 1954. *Logik und Existenz*, Meisenheim: Westkulturverlag Anton Hain.

Menut, A. D. and Denomy, A. J. 1941. ‘Maistre Nicole Oresme, Le livre du ciel et du monde, Text and commentary, Livre I’, *Mediaeval Studies*, **3**: 185–280.

Menut, A. D. and Denomy, A. J. 1942. ‘Maistre Nicole Oresme, Le livre du ciel et du monde, Text and commentary (continued), Livre II’, *Mediaeval Studies*, **4**: 159–297.

Menut, A. D. and Denomy, A. J. 1943. ‘Maistre Nicole Oresme, Le livre du ciel et du monde, Text and commentary (continued), Livres III et IV’, *Mediaeval Studies*, **5**: 167–333.

Menut, A. D. and Denomy, A. J. 1969. *Nicole Oresme, Le Livre du ciel et du monde*, Madison, WI: University of Wisconsin Press.

Moore, M. S. 1998. ‘Liberty and supererogation’, *Jahrbuch für Recht und Ethik*, **6**: 111–143.

Oresme, N. 2019. *Aristote, De Caelo et mundo traduit de latin en français par Nicole Oresme*. Bibliothèque nationale de France (BnF), Ms. franç. 1082, ff. 1a–209c, available online: http://archivesetmanuscrits.bnf.fr/ark:/12148/cc78170k (accessed on 16 July 2019).

Parsons, T. 2017. ‘The traditional square of opposition’, in: E. N. Zalta, ed., *Stanford Encyclopedia of Philosophy (Summer 2017 Edition)*, Stanford, CA: CSLI.

Patar, B. 1995. *Nicolai Oresme, Expositio et Quaestiones in Aristotelis “De Anima”*, Leuven: Peeters.

Read, S. 2012. ‘John Buridan’s theory of consequence and his octagons of opposition’, in: J.-Y. Béziau and D. Jacquette, eds., *Around and Beyond the Square of Opposition*, Berlin: Springer, 93–110.

Roelandt, K. 2016. *The Meaning of* Most*. Proportional and Comparative Interpretations in Dutch*, Utrecht: LOT Publications.

Schmidt, J. R. and Thompson, V. A. 2008. ' "At least one" problem with "some" formal reasoning paradigms', *Memory & Cognition*, **36**: 217–229.

Sesmat, A. 1951. *Logique II. Les Raisonnements. La syllogistique*, Paris: Hermann.

Seuren, P. and Jaspers, D. 2014. 'Logico-cognitive structure in the lexicon', *Language*, **90**: 607–643.

Smessaert, H. 2009. 'On the 3D visualisation of logical relations', *Logica Universalis*, **3**: 303–332.

Smessaert, H. and Demey, L. 2014. 'Logical geometries and information in the square of opposition', *Journal of Logic, Language and Information*, **23**: 527–565.

Smessaert, H. and Demey, L. 2015. 'Béziau's contributions to the logical geometry of modalities and quantifiers', in: A. Koslow and A. Buchsbaum, eds., *The Road to Universal Logic. Volume 1*, Berlin: Springer, 475–494.

Smessaert, H. and Demey, L. 2016. 'Visualising the Boolean algebra $\mathbb{B}_4$ in 3D', in: M. Jamnik, Y. Uesaka, and S. E. Schwartz, eds., *Diagrams 2016*, LNCS 9781, Berlin: Springer, 289–292.

Smessaert, H. and Demey, L. 2017a. 'The unreasonable effectiveness of bitstrings in logical geometry', in: J.-Y. Béziau and G. Basti, eds., *The Square of Opposition: A Cornerstone of Thought*, Berlin: Springer, 197–214.

Smessaert, H. and Demey, L. 2017b. 'The unreasonable effectiveness of bitstrings in logical geometry', *South American Journal of Logic*, **3**: 225–272.

Thijssen, J. M. M. H. 2004. 'The Buridan school reassessed. John Buridan and Albert of Saxony', *Vivarium*, **42**: 18–42.

Tierney, B. 2007. 'Obligation and permission: On a 'deontic hexagon' in Marsilius of Padua', *History of Political Thought*, **28**: 419–432.

Wybraniec-Skardowska, U. 2016. 'Logical squares for classical logic sentences', *Logica Universalis*, **10**: 293–312.